CAMBRIDGE
International Education

Cambridge International AS & A Level

MATHEMATICS **9709/52**

Paper 5 Probability and Statistics 1 **February/March 2025**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect point
Dep	Used to indicate DM0 or DM1

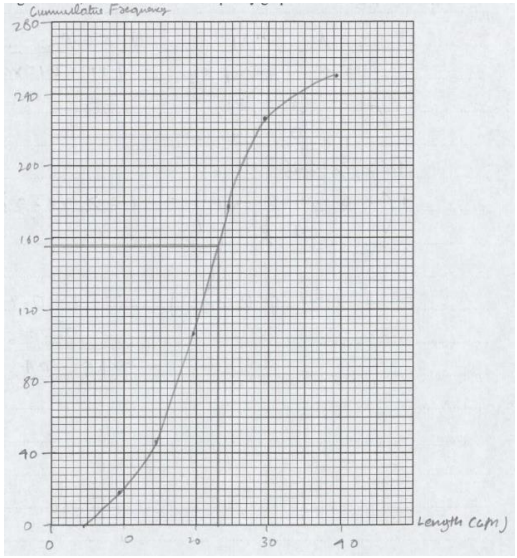
Annotation	**Meaning**
DM1	Dependent on the previous M1 mark(s)
FT	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
ISW	Ignore subsequent work
J	Judgement
JU	Judgement
M0	Method mark awarded zero
M1	Method mark awarded one
M2	Method mark awarded two
MR	Misread
O	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
PE	Judgment made by the PE
Pre	Premature approximation
SC	Special case

Annotation	Meaning
SEEN	Indicates that work/page has been seen
SF	Error in number of significant figures
	Correct point
TE	Transcription error
XP	Correct answer from incorrect working

Question	Answer	Marks	Guidance
1(a)	$\frac{1}{3}\times\frac{1}{4}\times\frac{4}{5}+\frac{1}{3}\times\frac{3}{4}\times\frac{1}{5}+\frac{2}{3}\times\frac{1}{4}\times\frac{1}{5}=\left[\frac{9}{60}=\right]\frac{3}{20}$ **AG**	**B1**	Order of coins must be consistent with question if not identified.
		1	
1(b)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>P($X = x$)</td><td>$\frac{24}{60}, \frac{8}{20}, \frac{2}{5}$ 0.4</td><td>$\frac{26}{60}, \frac{13}{30},$ 0.433</td><td>$\frac{9}{60}, \frac{3}{20},$ 0.15</td><td>$\frac{1}{60},$ 0.0167</td></tr></table>	**B1**	Table with correct values of x and at least two correct non-zero probabilities.
		B1	One **more** correct non-zero probability linked with correct x value, need not be in table if clearly identified, accept unsimplified (total of 3 correct probabilities).
		B1	4 correct probabilities linked with the correct outcomes, may not be in table. Decimals correct to at least 3SF. **SC1** for 4 or more probabilities summing to 1 placed in a probability distribution table.
		3	
1(c)	[Var(X) =] $\frac{(0^2\times 24+)\ 1^2\times 26+2^2\times 9+3^2\times 1}{60}-\left(\frac{47}{60}\right)^2$ $=\frac{1\times 26+4\times 9+9\times 1}{60}-\left(\frac{47}{60}\right)^2$	**M1**	Appropriate variance formula using $(E(X))^2$ value. FT *their* table with **3 or more** probabilities ($0 < p < 1$) which need not sum to 1, with an expression no more evaluated than in **bold**. FT acceptable at the bold partially evaluated stage with *their* probabilities.
	$= \frac{2051}{3600}, 0.570$	**A1**	$0.5695 < \text{Var}(X) \leqslant 0.570$. If M0 scored, **SC1** for $\frac{2051}{3600}, 0.570$ WWW Note: 0.57 without more accurate previous value penalised as 2SF.
		2	

Question	Answer	Marks	Guidance
2(a)	$[0.55\times0.3\times0.15\times3!=]0.1485, \frac{297}{2000}$	**B1**	Accept $\frac{148500}{1000000}$, condone 0.149.
		1	
2(b)	**Method 1**		
	[1 – P(10, 11, 12) =] $1 - \{^{12}C_{10}\, 0.55^{10}\, 0.45^{2} + {}^{12}C_{11}\, 0.55^{11}\, 0.45 + 0.55^{12}\} =$ [1 – (0.0338529 + 0.0075229 + 0.0007662) =]	**M1**	One term of the form $^{12}C_x\,(p)^x(1-p)^{12-x}$, $0 < p < 1$, $x \neq 0$ or 12.
		A1	Correct unsimplified expression, no terms omitted leading to final answer. Condone omission of last bracket '}' only.
	= 0.958	**B1**	$0.9575 < p \leq 0.958$.
	Method 2		
	[P(0,1,2,3,4,5,6,7,8,9) =] $0.45^{12} + {}^{12}C_1\, 0.55^{1}\, 0.45^{11} + \ldots + {}^{12}C_9\, 0.55^{9} 0.45^{3}$	**M1**	One term of the form $^{12}C_x\,(p)^x(1-p)^{12-x}$, $0 < p < 1$, $x \neq 0$ or 12.
		A1	Correct unsimplified expression, no more than 7 'middle' terms omitted leading to final answer.
	= 0.958	**B1**	$0.9575 < p \leqslant 0.958$.
		3	

Question	Answer	Marks	Guidance
2(c)	[Mean = $140\times0.15=$] 21 [Variance = $140\times0.15\times0.85=$] 17.85	**B1**	21 and 17.85 (or $17\frac{17}{20}$) seen, allow unsimplified. May be in standardisation formula. ($\sigma=\sqrt{17.85}, 4.224926\ldots$ to at least 4SF implies correct variance). Withhold mark if variance clearly identified as standard deviation, condone N(21, $\sqrt{17.85}$) **if standardisation formula correct** *or* variance/standard deviation correctly stated as well.
	$P(X > 24) = P\left(Z > \frac{24.5-21}{\sqrt{17.85}}\right)$	**M1**	Substituting *their* μ and *their* σ into the ± standardisation formula (any number for 24.5), allow σ^2 or $\sqrt{\sigma}$.
		M1	Use continuity correction 23.5 or 24.5 in *their* standardisation formula. Note: If no working $\pm\left(\frac{3.5}{\sqrt{17.85}}\right)$ *or* $\pm\left(\frac{3.5}{4.225}\right)$ seen gains M2 BOD.
	[$P(Z > 0.8284) = 1-\Phi(0.8284)$] 1 – 0.7961	**M1**	Appropriate area Φ, from final process, must be a probability. May be implied by a sketch of the required probability area. Note: correct final answer implies this M1. Expect final answer < 0.5.
	= 0.204	**A1**	Final answer AWRT.
		5	

Question	Answer	Marks	Guidance
3(a)	<table><tr><td></td><td>< 9.5</td><td>< 14.5</td><td>< 19.5</td><td>< 24.5</td><td>< 29.5</td><td>< 39.5</td></tr><tr><td>CF</td><td>18</td><td>46</td><td>106</td><td>178</td><td>226</td><td>250</td></tr></table> 	**B1**	At least 4 of 46, 106, 178, 226, 250 cumulative frequencies correct. May be implied by accurate plotting if scale suitable. May be by data table.
		B1	Linearly scaled axes correctly labelled cumulative frequency (cf) (from 0 to 250), length (oe) and cm (from 5 to 39.5) – or a suitable title, with at least 3 values identified on each. Axes can be the other way round. Axes must be more than 50% of grid.
		M1	At least 4 points correctly plotted at class upper end points (9.5, 14.5, 19.5, 24.5, 29.5, 39.5) on scaled axes.
		A1	All points plotted correct, curve drawn (within tolerance) and joined to (4.5,0) and not going above 250 vertically within range. A0 if straight line segments used.
		4	
3(b)	[250 × 0.62 = 155]	**M1**	Clear indication of use of graph at cf 155 is required.
	Line drawn from 155 on cf axis to meet graph at l = 23	**A1FT**	Must be an increasing cf graph. Expect an answer in the range $22.5 \leqslant l \leqslant 23.5$ from correct graph.
		2	

Question	Answer	Marks	Guidance
3(c)	Midpoints 7, 12, 17, 22, 27, 34.5	**B1**	At least 5 correct midpoints seen, may be unsimplified, may be in calculation, may be by data table.
	Mean = $\frac{7\times18+12\times28+17\times60+22\times72+27\times48+34.5\times24}{250}$	**M1**	Correct unsimplified mean formula using *their* 6 midpoints (not upper bound, lower bound, upper limits, lower limits, cw, fd, f or cf and must be within class) condone 1 error. If midpoints correct accept $\frac{5190}{250}$ or $\frac{126+336+1020+1584+1296+828}{250}$.
	= 20.76	**A1**	Accept $20\frac{19}{25}, 20\frac{38}{50}$ or $20\frac{76}{100}$, not improper fraction If M1 withheld, **SC1** for 20.76 oe WWW.
		3	

Question	Answer	Marks	Guidance
4(a)	P(WWW) = $\frac{8}{16}\times\frac{7}{15}\times\frac{6}{14}\left[=\frac{336}{3360},\frac{1}{10}\right]$ or $\frac{{}^{8}C_{3}}{{}^{16}C_{3}}$ P(BBB) = $\frac{5}{16}\times\frac{4}{15}\times\frac{3}{14}\left[=\frac{60}{3360},\frac{1}{56}\right]$ or $\frac{{}^{5}C_{3}}{{}^{16}C_{3}}$ P(SSS) = $\frac{3}{16}\times\frac{2}{15}\times\frac{1}{14}\left[=\frac{6}{3360},\frac{1}{560}\right]$ or $\frac{{}^{3}C_{3}}{{}^{16}C_{3}}$	**M1**	1 outcome seen as the product of 3 fractions with 8, 7, 6 or 5, 4, 3 or 3, 2, 1 as numerators and 16, 15, 14 or 16, 16, 16 as denominators Or 1 outcome correct in terms of combinations.
		M1	Sum of 3 correct identified scenarios (may be identified by correct unsimplified numerator values).
	$\frac{402}{3360}, \frac{67}{560}, 0.120$	**A1**	$0.1195 < p \leqslant 0.120$.
		3	

Question	Answer	Marks	Guidance
4(b)	3 cases to consider: WBW, WBB, WBS P(WBW) = $\frac{8}{16}\times\frac{5}{15}\times\frac{7}{14}\times3\left(=\frac{840}{3360},\frac{1}{4}\right)$ P(WBB) = $\frac{8}{16}\times\frac{5}{15}\times\frac{4}{14}\times3\left(=\frac{480}{3360},\frac{5}{42}\right)$ P(WBS) = $\frac{8}{16}\times\frac{5}{15}\times\frac{3}{14}\times6\left(=\frac{720}{3360},\frac{3}{14}\right)$ **Or** P(WBW) = $\frac{{}^{8}C_{2}\times{}^{5}C_{1}}{{}^{16}C_{3}}\left(=\frac{140}{560}\right)$ P(WBB) = $\frac{{}^{8}C_{1}\times{}^{5}C_{2}}{{}^{16}C_{3}}\left(=\frac{80}{560}\right)$ P(WBS) = $\frac{{}^{8}C_{1}\times{}^{5}C_{1}\times{}^{3}C_{1}}{{}^{16}C_{3}}\left(=\frac{120}{560}\right)$	**M1**	One outcome seen as the product of 3 fractions with correct numerators and n, $(n-1)$, $(n-2)$ **only** as denominator, where $8 \leqslant n \leqslant 16$ (condone omission of ×3 or ×6) Or 1 outcome correct in terms of combinations with ${}^{m}C_{3}$ as denominator where $8 \leqslant m \leqslant 16$. Must be a probability, no additional 'divisions' leading to final answer.
		A1	1 identified outcome fully correct (accept unsimplified).
		M1	Sum of 3 correctly identified scenarios (may be identified by correct unsimplified numerator values).
	$\frac{2040}{3360},\frac{17}{28},0.607$	**A1**	If 1 or more M mark not scored, **SC1** for $\frac{2040}{3360},\frac{17}{28},0.607$ WWW.
	Method 2 $1-\{P(WSS)+P(WWS)+P(WWW)+P(BSS)+P(BBS)+P(BBB)+P(SSS)\}$		
	P(WSS) = $\frac{8}{16}\times\frac{3}{15}\times\frac{2}{14}\times3\left(=\frac{144}{3360},\frac{3}{70}\right)$ P(WWS) = $\frac{8}{16}\times\frac{7}{15}\times\frac{3}{14}\times3\left(=\frac{504}{3360},\frac{3}{20}\right)$ P(WWW) = $\frac{8}{16}\times\frac{7}{15}\times\frac{6}{14}\left(=\frac{336}{3360},\frac{1}{10}\right)$	**M1**	**Two** outcomes seen as the product of 3 fractions with correct numerators and n, $(n-1)$, $(n-2)$ **only** as denominator, where $8 \leqslant n \leqslant 16$ (condone omission of ×3). Attempt at $1-p$ must be present. Must be a probability, no additional 'divisions' leading to final answer.
		A1	2 identified outcomes fully correct (accept unsimplified).

Question	Answer	Marks	Guidance
4(b)	P(BSS) = $\frac{5}{16}\times\frac{3}{15}\times\frac{2}{14}\times 3\left(=\frac{90}{3360},\frac{3}{112}\right)$ P(BBS) = $\frac{5}{16}\times\frac{4}{15}\times\frac{3}{14}\times 3\left(=\frac{180}{3360},\frac{3}{56}\right)$ P(BBB) = $\frac{5}{16}\times\frac{4}{15}\times\frac{3}{14}\left(=\frac{60}{3360},\frac{1}{56}\right)$ P(SSS) = $\frac{3}{16}\times\frac{2}{15}\times\frac{1}{14}\left(=\frac{6}{3360},\frac{1}{560}\right)$	**M1**	1 – sum of 7 correctly identified scenarios (may be identified by correct unsimplified numerator values).
	$\frac{2040}{3360},\frac{17}{28}$	**A1**	If 1 or more M mark not scored, **SC1** for $\frac{2040}{3360},\frac{17}{28},0.607$ WWW.
		4	

Question	Answer	Marks	Guidance
5(a)	[P(56 < X < 75) =] P($\frac{56-65.8}{9.6} < Z < \frac{75-65.8}{9.6}$) $\left[= P\left(-1.0208 < Z < 0.9583\right)\right]$	**M1**	Use of ±standardisation formula once with 65.8, 9.6 and either 56 or 75. No continuity correction, not σ^2, not $\sqrt{\sigma}$.
	[$\Phi\left(0.9583\right) + \Phi\left(1.0208\right) - 1$] = 0.8309 + 0.8463 – 1 or 0.8309 – (1 – 0.8463) or 0.8309 – 0.1537 or (0.8309 – 0.5) + (0.8463 – 0.5) or 0.3309 + 0.3463	**M1**	Appropriate probability area Φ, from final process. Must be a probability. (expect > 0.5).
	= 0.677	**A1**	AWRT. If 1 or more M mark not awarded, **SC1** for final answer 0.677 AWRT WWW.
		3	
5(b)	$\left[P\left(Z < \frac{59.1-72.4}{\sigma}\right) = 0.10\right]$	**B1**	1.282 or – 1.282 seen cao (critical value).
	$\frac{59.1-72.4}{\sigma} = -1.282$	**M1**	±standardisation formula with 59.1, 72.4, σ equating to a z-value (not 0.1, 0.9, 0.5398, 0.4602, 0.8159, 0.1841). Condone continuity correction of ±0.05, not σ^2 and not $\sqrt{\sigma}$. Condone $\pm\frac{13.3}{\sigma} = -1.282$.
	$\sigma = 10.4$	**A1**	AWRT. Signs must be consistent throughout. If M1 not awarded, **SC1** $\sigma = 10.4$ WWW.
		3	

Question	Answer	Marks	Guidance
5(c)	$\left[(0.9)^4(0.1)=\right]0.0656[1]$	**B1**	
		1	
5(d)	**Method 1**		
	[P(X < 5) =] $1-(0.9)^4$	**M1**	$1-(0.9)^d$ d = 4, 5.
	= 0.344	**A1**	0.3439.
	Method 2		
	[P(X < 5) =] $0.1+(0.1)(0.9)+(0.1)(0.9)^2+(0.1)(0.9)^3$ or $1-((0.1)(0.9)^4+(0.1)(0.9)^5+(0.1)(0.9)^6+(0.9)^7)$	**M1**	$0.1+(0.1)(0.9)+(0.1)(0.9)^2+(0.1)(0.9)^3\left[+(0.1)(0.9)^4\right]$ or $1-(\left[(0.1)(0.9)^4\right]+(0.1)(0.9)^5+(0.1)(0.9)^6+(0.9)^7)$.
	= 0.344	**A1**	0.3439.
		2	

Question	Answer	Marks	Guidance
6(a)	$\left[\frac{10!}{2!3!2!}=\right]151200$	**B1**	CAO.
		1	
6(b)	**Method 1** Total – Ys together		
	$\frac{8!}{2!2!}-\frac{7!}{2!}=$ [10080 – 2520 =]	**M1**	$\frac{8!}{2!a!c!}-\frac{7!}{a!b!c!}$, a = 1, 2 b = 1, 2 c = 1, 3.
	7560	**A1**	
		2	
	Method 2 3 Bs treated as a single unit ^ ^ ^ ^ BBB ^ and Ys inserted		
	$\frac{6!}{2!}\times {}^{7}\mathrm{C}_{2}$ or $\frac{6!}{2!}\times\frac{7\times6}{2}$ or $\frac{6!}{2!}(6+5+4+3+2+1)$ [360 × 21 =]	**M1**	$\frac{6!}{d!}\times {}^{7}\mathrm{C}_{2}$ or $\frac{6!}{d!}\times {}^{7}\mathrm{P}_{2}$ or $\frac{6!}{d!}\times\frac{{}^{7}\mathrm{P}_{2}}{2}$ or $\frac{6!}{d!}\times\frac{7\times6}{e}$ or $\frac{6!}{d!}(6+5+4+3+2+1)$ d = 1, 2, 3 e = 1, 2.
	7560	**A1**	
		2	

Question	Answer	Marks	Guidance
6(c)	**Method 1**		
	$\frac{8!}{2!3!}\times 5$	***M1**	$\frac{8!}{b!c!}\times d$ **,** b = 1,2, c = 1,3 **and** $b \neq c$, $d \geq 1$.
		DM1	Multiply by 5.
	16 800	**A1**	
	Method 2		
	$\frac{{}^{8}\mathrm{P}_{4}\times 5!}{2!3!}$	***M1**	$\frac{{}^{8}\mathrm{P}_{4}}{b!c!}\times d$ **,** b = 1,2, c = 1,3 **and** $b \neq c$, $d \geq 1$.
		DM1	Multiply by 5!.
	16 800	**A1**	
		3	

Question	Answer	Marks	Guidance
6(d)	**Method 1**		
	Y R B _ $2 \times 2 \times {}^3C_1 \times {}^3C_1 = 36$ Y R R B $2 \times 1 \times {}^3C_1 = 6$ Y R _ _ $2 \times 2 \times {}^3C_2 = 12$ Y R R _ $2 \times 1 \times {}^3C_1 = 6$	**M1**	One correct identified unsimplified expression ($^3C_1 \neq {}^3C_2$).
		B1	Correct outcome/value for 2 clearly identified scenarios, accept unsimplified WWW.
		M1	Sum of 4 correct identified scenarios.
	60	**A1**	
	Method 2		
	Y R _ _ ${}^5C_3 \times {}^3C_1 \times {}^2C_1$	**M1**	${}^5C_3$ seen with YR^^ identified.
		M1	${}^5C_3 \times a$, $a = 2, 3, 6$.
		B1	${}^5C_3 \times {}^3C_1 \times {}^2C_1$ *or* ${}^5C_3 \times 3 \times 2$.
	60	**A1**	
		4	